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CO-CURRENT FLOW OF AIR AND WATER FROM A RESERVOIR INTO A SHORT HORIZONTAL PIPE

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Abstract—Co-current air–water flow from a reservoir into and along a short horizontal pipe is studied at flowrates such as to give stratified flow and to observe the transition to spray and slug flow. Particularly noteworthy is that a square-cut entry as opposed to a bell-mouth entry increased the intensity of spray and suppressed the formation of large waves.

The hydraulic engineer's theory for the discharge rate of water alone as a function of water level in the reservoir can only be extended in agreement with experiment for small air flowrates. Such a theory asserts that the flow in the pipe is critical in the sense that a small disturbance is stationary. In practice, at higher air flowrates the flow becomes supercritical and the critical condition occurs upstream of the entry at a control point. Examination of the control point in a slowly converging flow suggests the relationship

$$h_1 = 0.862\,(F_H^{0.4} - F_L^{0.4})$$

between the level in the reservoir and the flowrates of the two phases. Surprisingly, no modification to this relationship is found to be necessary with respect to the present data for an 84 mm bore pipe and previous data for a 20 mm bore pipe.

For a 6 mm bore pipe the difference between previous experimental results and theory is equivalent to a constant level difference of 2.5 mm.

Key Words: air–water flow, co-current flow, horizontal pipe entry

1. INTRODUCTION

The mechanism of water flow from a reservoir over a broad-crested weir or into a short horizontal pipe or culvert, which does not run full, has been known for many years. Chow (1981) gives a summary. The present investigation concerns adding a co-current flow of air for the case of a pipe. Without air flow the theory assumes no energy losses and maximizes the volumetric flowrate for a given level in the reservoir. The result is that the downstream flow is critical in the sense that a small surface disturbance is stationary. It will be shown that the theory is readily extended to include the influence of air flow, which also suffers no losses, by maximizing the water flowrate for a given air flowrate and level in the reservoir. Again, the result is that downstream flow is critical but it is also found that, if a water flowrate is set and the reservoir level is allowed to vary, increase of the air flowrate causes the downstream water level in the pipe to increase. This is at variance with experimental observation and, indeed, the downstream flow becomes supercritical when a certain air flowrate is exceeded.

A new theoretical approach is needed and this is obtained by considering two-phase stratified flow along a horizontal slowly converging channel or tube to a sink. Far upstream velocities will be vanishingly small and the interface will be horizontal. Proceeding towards the sink, the flow will at first be subcritical but must become supercritical before the sink is attained. It will be shown by an examination of the interface level gradient, that the gradient can only be finite and continuous when the flow passes through the critical state if a certain condition is met and this will determine the interface levels throughout the system. The position where critical flow obtains is called the control point.

One can now consider a case where the converging channel or tube is terminated by a horizontal channel or tube of constant cross-section, which is called the offtake. If the offtake dimensions are such that the flow there would be subcritical, the control point does not exist and a return must be made to the original kind of theory described in the first paragraph, which will set the flow to be critical in the offtake. If the offtake dimensions are smaller and such that the flow there would be supercritical, the control point will determine interface levels throughout the system and the expected supercritical flow will occur on the offtake, as long as the flow remains stratified. However, the theory only requires the flow to be stratified up to the control point and this allows it to be tested against data, for which stratified flow breaks down before the offtake is attained.

The problem is to change considerations from the slowly converging channel or tube to a system in which there is an interface in a large reservoir and the two phases are discharged through an offtake in a vertical wall. It may be noted that the theory for the slowly converging system does not explicitly consider the rate of convergence and, in this sense, the change offers no difficulty. However, the theory assumes that the velocity of either phase is horizontal and uniform over a vertical plane and, for this reason, it is to be expected that the analytical results obtained from the slowly converging system will require some adjustment for application to the system with an offtake in a vertical wall. In the event and rather surprisingly, it will be seen that experimental data indicate that no modification is needed.

At this point it should be noted that the concept of a control point is not new. Dressler (1949) used the properties of the control point to describe the shape of a roll-wave in open channel flow. Bryant & Wood (1976) considered the control point in slowly converging flow and Gardner (1980) used it with respect to the flow of stably stratified fluids which were being added along the length of a horizontal channel. Otherwise, theory on the discharge of fluids from a reservoir through an offtake in a vertical wall has only be concerned with the critical flowrate of one phase which will start to entrain the other. Craya's (1949) theory was confirmed experimentally by Gariel (1949) and much later by Smoglie & Reimann (1986).

The experimental work to be reported concerns discharge of air and water from a reservoir through a short horizontal pipe of 84 mm bore and with various forms of entry to the pipe. Flowrates were such that the flow in the pipe was stratified, though transition to spray and large wave or slug flow were observed. The conditions for these transitions will be described first and then the correlation between the level in the reservoir and the flowrates will be presented. Smoglie & Reimann (1986) also measured the level in the reservoir with respect to the flowrates but they used much higher flowrates, such that the flow in the offtake would not have been stratified. Their data will be found to be in substantial agreement with the theory. Smoglie & Reimann (1986) used offtakes with a bore from 6 to 20 mm and used air and water at pressures up to 5 b. The present work was carried out at atmospheric pressure.

2. THE RIG AND MEASUREMENTS

The rig is illustrated in figure 1. The reservoir was a cylindrical Perspex vessel of 430 mm dia and 1200 mm height. Air and water entered the top and bottom respectively through 50 mm dia pipes and 150 mm dia baffles were placed 75 mm in front of each entry. The baffles contained 7 mm dia holes to give 14% open area and the whole arrangement prevented any appreciable disturbance of an air–water interface about halfway up the reservoir.

The discharge pipe was a 84 mm dia × 590 mm long Perspex tube set in the reservoir wall halfway up the height. One type of entry is shown in figure 1. It is called the PWR entry because it models the entry from the upper plenum of a PWR reactor to the so-called hot leg. It is noted that it is flush with the reservoir wall, following its curvature, and has a champfer.

Two other types of entry are shown in figure 2. They are the square-cut, which projected into the reservoir and which extended the pipe by 50 mm, and the bell-mouth, which extended the pipe a further 35 mm when it was attached to the square-cut entry.

The air discharged to atmosphere from the discharge pipe and the water fell freely into a water tank for recirculation. There were two measuring stations along the discharge pipe, as shown in figure 1. At each there was a pressure tap at the top and bottom of the tube which were connected to measure the water level with respect to the bottom of the discharge pipe. The top tapping was also connected to a water manometer to measure pressure.

Pressure taps near the bottom and top of the reservoir were connected to measure the water level in the reservoir relative to the bottom of the discharge tube. The top tapping was connected to a water manometer to measure the air pressure. Air and water flowrates to the reservoir were metered by orifice plates.

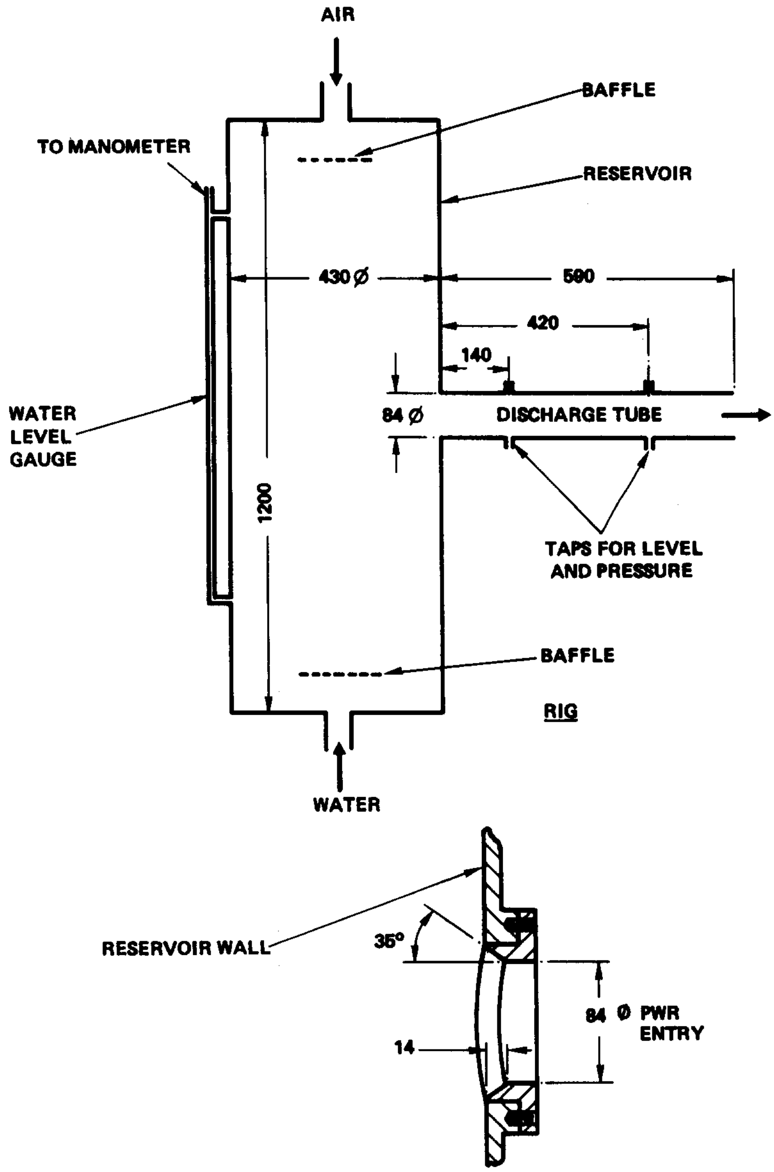

Figure 1. Rig and PWR entry.

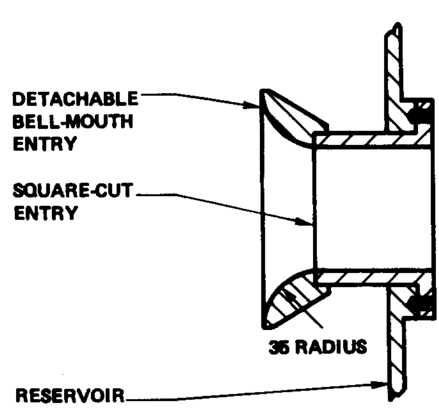

Figure 2. Square-cut and bell-mouth entries.

3. VISUAL OBSERVATIONS AND FLOW REGIMES

With no air flowrate, the water fell from its level in the reservoir into the discharge tube in the expected fashion and a vena contracta was evident. An undular hydraulic jump formed, indicating that the entrances were not ideal. Application of only a small air flowrate removed the waves of the hydraulic jump, while the interface remained essentially smooth.

Further description will be given with reference to the flow regime maps of figures 3–5 for the three different entries. They are plotted in the coordinates of

$$F_{\mathrm{H}} = \left(\frac{\rho_{\mathrm{H}}}{\Delta\rho g D'}\right)^{\frac{1}{2}} u_{\mathrm{HS}}, \quad F_{\mathrm{L}} = \left(\frac{\rho_{\mathrm{L}}}{\Delta\rho g D'}\right)^{\frac{1}{2}} u_{\mathrm{LS}} \qquad [1]$$

where subscripts $_{\mathrm{H}}$ and $_{\mathrm{L}}$ refer to the heavy or water phase and the light or air phase; ρ is the density, $\Delta\rho$ is the density difference between the phases, g is the acceleration due to gravity, D' is the tube diameter and u_{S} is the superficial velocity, which is the flowrate divided by the cross-sectional area of the tube.

The flow regime boundaries were determined by setting a water flowrate and then increasing the air flowrate in small steps. The system was observed by looking through the Perspex reservoir into and along the discharge tube when the occurrence of the boundaries could be determined accurately. The first boundary that could be defined for the bell-mouth and PWR entries but not for the square-cut entry was the occasional and erratic production of waves with a height of up to about 10 mm, which travelled down the discharge tube from near the entrance. It occurred at values of F_{H} from 0.28 to 0.45 and usually on a base of a smooth interface. Exceptions are the two data points at values of F_{H} near 0.3 in the case of the PWR entry. Here the waves appeared on a base of a pebbled interface, which may have obscured their formation at lower values of F_{L} than shown in figure 4.

Quite generally, for values of $F_{\mathrm{H}} \lesssim 0.45$, raising the air flowrate to a value, which could not be defined precisely, caused the interface to take on a pebbled appearance, which became more violent as the air flowrate was further increased. Where small occasional waves on a smooth interface occurred, these may have been a precursor of this pebbling. The pebbling may have occurred earlier with the square-cut entry and, certainly, it was more vigorous.

An air flowrate was reached at which water drops, through few in number, could be seen continuously in the air flow. This is recorded as a flow regime boundary.

The small occasional waves, for the values of F_{H} at which they occurred and only for the bell-mouth and PWR entries, persisted throughout all of this, even when spray formed and became

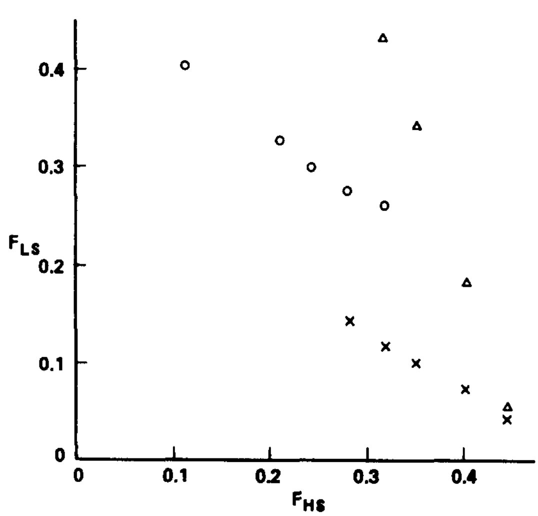

Figure 3. Bell-mouth entry flow regime boundaries: ×, small occasional waves; ○, drops in air; △, large waves to hit top of tube.

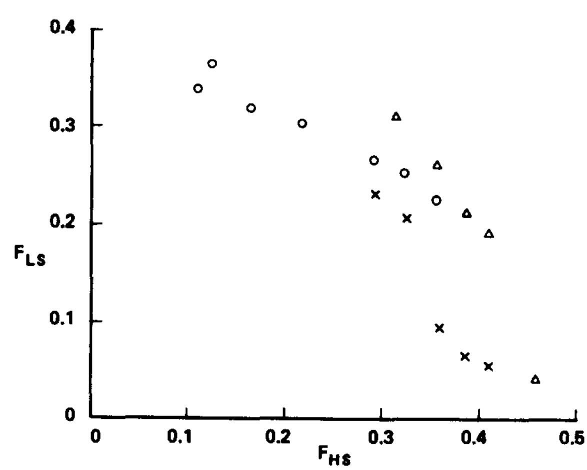

Figure 4. PWR entry flow regime boundaries. See figure 3 for symbols.

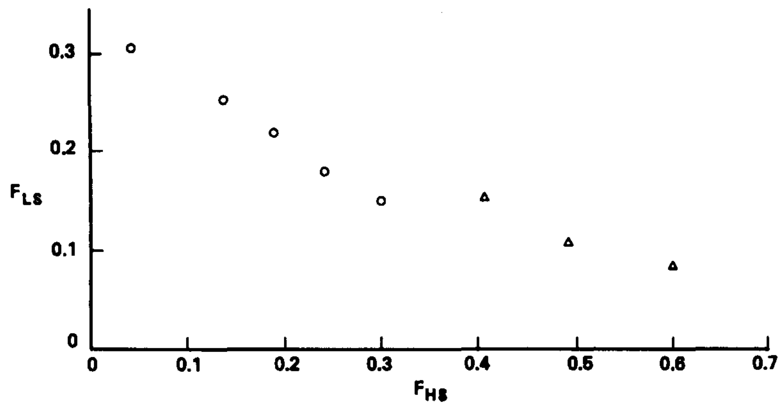

Figure 5. Square-cut entry flow regime boundaries. See figure 3 for symbols.

more intense with increased air flowrate. It did appear that, for some intermediate air flowrates, the waves became weaker but, finally, they grew until the occasional wave hit the top of the tube and obscured the rim at its end. This is recorded as a flow regime boundary and only occurred over the range of parameters indicated in figures 3–5. These waves occurred erratically but sometimes came in sets of two or three.

It is noted from figure 5 that waves hitting the top of the tube occurred at substantially higher water flowrates in the case of the square-cut entry compared to the other two. It was also observed that these waves tended to disappear at higher air flowrates than shown by the boundary, though with the bell-mouth and PWR entries they tended to increase in number and vigour at all air flowrates up to the maximum attainable at $F_L = 0.5$. The marked difference between the flow regime map for the square-cut entry in figure 5 and for the other two entries in figures 3 and 4 seemed to arise from the greater disturbance created by the square-cut entry. This led to pebbling of the interface at lower air flowrates and more intense spray production. The impression was gained that the greater amount of spray robbed the system of its ability to produce waves.

A further observation was that, for any entry and, at least when pebbling had set in, the flowrate seemed to settle down very rapidly in the discharge tube, though there was a measurable reduction in the water level from the first to the second measuring station. Also, the water level fell with increased air flowrate. Since the flowrate was near critical without air flow, as exhibited by the undular hydraulic jump, it must have become supercritical with an air flowrate imposed. Calculations showed this to be true but it was shown more dramatically by using a block of wood held to partially dam the end of the discharge tube. With zero or a small air flowrate a wave was sent back the discharge tube but, otherwise, there was no influence on the upstream water depths. The water simply hit the block and spilt over it, together with the air.

4. CORRELATION OF FLOW REGIME BOUNDARIES

4.1. Onset of spray formation

It is common to use a Kutateladze number, defined by

$$\mathrm{Ku} = \frac{\rho_L^{\frac{1}{2}} u_L}{(\Delta \rho g \sigma)^{\frac{1}{4}}}, \qquad [2]$$

to define the onset of spray formation; u_L is the air velocity, which is calculated from a knowledge of the air flowrate and the water depth and σ is the surface tension. The data points in figure 3 for the bell-mouth give Ku = 2.53 with a maximum error of 3%; the data points for the PWR entry give Ku = 2.51, with a maximum error of 6%; and the data points for the square-cut entry give Ku = 2.15, with a maximum error of 12%. The water level at the first measuring station was used to calculate u_L but use of the level at the second station only reduced the average value of Ku by 7% for all entries.

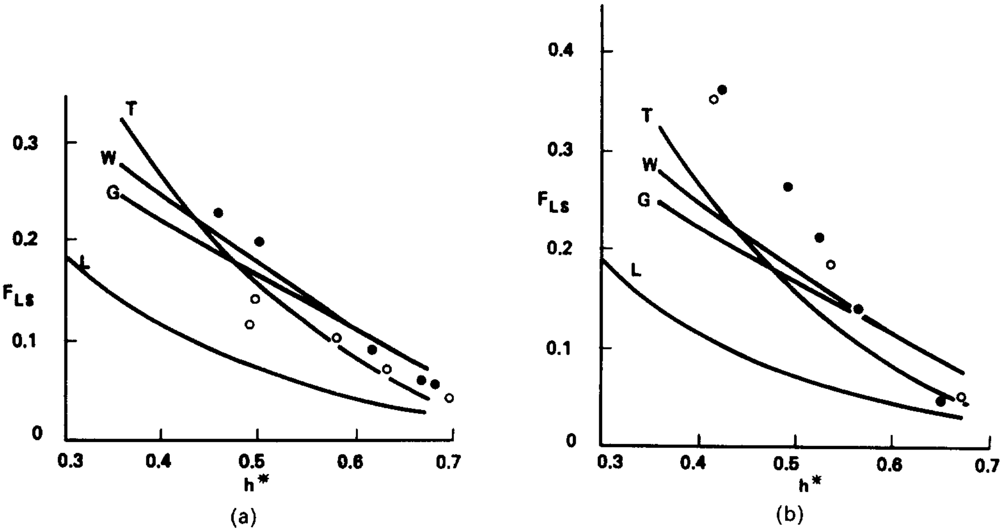

Figure 6. Wave flow regime boundaries: (a) occasional small waves; (b) large waves to hit top of tube. ○, Bell-mouth entry; ●, PWR entry; L, Lin & Hanratty (1986) experiment; G, Gardner (1979) theory; W, Wallis & Dobson (1973) theory; T, Taitel & Dukler (1976) theory.

4.2. Wave flow regime boundaries

Theories concerned with the production of waves, other than capillary waves, in two-phase stratified flow usually relate F_L to the voidage or the water depth divided by the tube diameter, h^*. The theoretical or semi-theoretical predictions of Wallis & Dobson (1973), Taitel & Dukler (1976) and Gardner (1979) for the onset of slugging or intermittent flow are given in the coordinates of F_L vs h^* in figure 6(a, b). Lin & Hanratty (1986) have also given a theory, which is in substantial agreement with their experimental results, but the curve through their results is given in figure 6(a, b).

Figure 6(a) gives the present experimental results for the onset of occasional small waves. The two data points for the PWR entry for h^* between 0.45 and 0.5 are suspect for the reason given in the last section that they occurred on the base of a pebbled interface, which may have obscured the formation of the waves at smaller values of F_L. Otherwise, the data fall a little below the theories of Wallis & Dobson (1973) and Gardner (1979) and well above the data of Lin & Hanratty (1986) but follow their trend.

Figure 6(b) gives the experimental results for large waves hitting the top of the tube. It is noticed that results for the bell-mouth and PWR entries agree with each other, though comparison of figures 3 and 4 shows this agreement was not obvious when the flow regime boundary was plotted in terms of F_L vs F_H. The data lie above all the theoretical curves for $h^* < 0.55$. The reason may simply be that the theories predict the onset of large waves which do not necessarily hit the top of the tube but waves of smaller amplitude will be needed for this purpose when h^* is large.

A further point is that Taitel & Dukler (1976) assumed for the purpose of constructing a flow regime map that large waves would not be produced for $h^* < 0.5$. Figure 6(b) shows that they are produced for values of h^* as small as 0.4.

5. THEORY OF FLOW INTO AN OFFTAKE

5.1. Extension of hydraulic engineers' theory

Figure 7 illustrates the system. Interface levels h' are measured upwards from the centreline of the offtake: p is pressure at the interface, u is velocity and D_2' is the characteristic dimension of the offtake, which is the diameter in the case of a pipe; subscripts 1 and 2 denote conditions in the reservoir and in the offtake and subscripts $_H$ and $_L$ denote heavy and light phases. Conservation of energy for both phases with zero velocities in the reservoir gives

$$\frac{\rho_H u_{H2}^2}{2} + p_2 + \rho_H g h_2' = p_1 + \rho_H g h_1' \qquad [3]$$

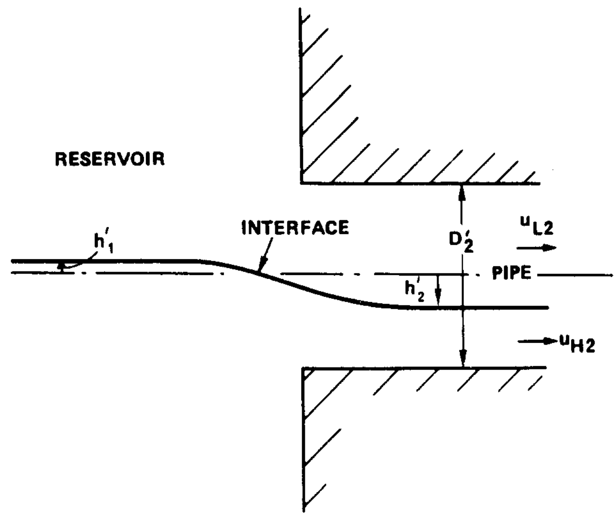

Figure 7. Sketch defining variables.

and

$$\frac{\rho_L u_{L2}^2}{2} + p_2 + \rho_L g h_2' = p_1 + \rho_L g h_1' \qquad [4]$$

where ρ is the density. Superficial velocities, denoted by subscripts, are given by

$$u_{HS2} = A_2 u_{H2}, \quad u_{LS2} = (1 - A_2) u_{L2} \qquad [5]$$

where A_2 is the fractional area occupied by the heavy phase in the offtake.

Pressures are eliminated between [3] and [4] and substitutions for the velocities are made from [5] to achieve

$$\frac{F_H^2}{A_2^2} - \frac{F_L^2}{(1 - A_2)^2} = 2(h_1 - h_2) \qquad [6]$$

where

$$F_H^2 = \frac{\rho_H u_{HS2}^2}{\Delta\rho g D_2'}, \quad F_L^2 = \frac{\rho_L u_{LS2}^2}{\Delta\rho g D_2'}, \qquad [7]$$

$$h = \frac{h'}{D_2'} \qquad [8]$$

and

$$\Delta\rho = (\rho_H - \rho_L).$$

Following hydraulic engineering concepts, not only is h_1 set constant but F_L is also kept constant. Then h_2 is varied to maximize F_H. The result is

$$\frac{F_H^2}{A_2^3} + \frac{F_L^2}{(1 - A_2)^3} = \frac{dh_2}{dA_2}. \qquad [9]$$

For the case of a pipe with a circular cross-section, [9] becomes

$$\frac{F_H^2}{A_2^3} + \frac{F_L^2}{(1 - A_2)^3} = \frac{\pi}{4W_2} \qquad [10]$$

where

$$W_2 = \frac{w_2'}{D_2'}; \qquad [11]$$

w_2' is the width of the interface. Equation [10] is recognized as the condition for a small disturbance to be stationary (Gardner 1977) and, therefore, the flow is critical in the offtake. Equation [10] defines h_2 for given values of F_H and F_L and then h_1 can be found from [6].

If one increases F_L from zero while keeping F_H constant, [10] shows that h_2 will increase monotonically. Experiment showed this not to be true in general.

5.2. Theory with a control point

This subsection will develop the theory for a two-dimensional system of a converging channel of depth H' leading to an offtake of depth H_2'. This will allow clarity of presentation and the theory for a tube is given in the appendix.

The horizontal direction is x and the differential energy equations are written as

$$\frac{\rho_H}{2}\frac{du_H^2}{dx} + \rho_H g \frac{dh'}{dx} + \frac{dp}{dx} = 0 \qquad [12]$$

and

$$\frac{\rho_L}{2}\frac{du_L^2}{dx} + \rho_L g \frac{dh'}{dx} + \frac{dp}{dx} = 0. \qquad [13]$$

The volumetric flux Q is constant for each phase, so that

$$Q_H = u_H\left(\frac{H'}{2} + h'\right), \quad Q_L = u_L\left(\frac{H'}{2} - h'\right). \qquad [14]$$

The pressure gradient is eliminated between [12] and [13], and [14] is used to eliminate velocities. Manipulation of the result gives

$$\left[1 - \frac{F_H^2}{\left(\frac{H}{2}+h\right)^3} - \frac{F_L^2}{\left(\frac{H}{2}-h\right)^3}\right]\frac{dh}{dX} = \left[\frac{F_H^2}{\left(\frac{H}{2}+h\right)^3} - \frac{F_L^2}{\left(\frac{H}{2}-h\right)^3}\right]\frac{dH}{dX} \qquad [15]$$

where F_H, F_L and h are given by [7] and [8] with the characteristic dimension changed to H_2' and

$$H = \frac{H'}{H_2'}, \quad X = \frac{x}{H_2'}. \qquad [16]$$

It is noted that the superficial velocity in the offtake is given by Q/H_2'.

The contents of the square brackets on the l.h.s. of [15] set equal to zero is the critical flow condition and, therefore, if that condition is obtained and the interface gradient is to be finite and continuous, the contents of the square brackets on the r.h.s. of [15] must simultaneously equal zero. Denote the control point by subscript c. Then

$$1 - \frac{F_H^2}{\left(\frac{H_c}{2}+h_c\right)^3} - \frac{F_L^2}{\left(\frac{H_c}{2}-h_c\right)^3} = 0 \qquad [17]$$

and

$$\frac{F_H^2}{\left(\frac{H_c}{2}+h_c\right)^3} = \frac{F_L^2}{\left(\frac{H_c}{2}-h_c\right)^3}. \qquad [18]$$

From [17] and [18],

$$H_c = 2^{1/3}(F_H^{2/3} + F_L^{2/3}) \qquad [19]$$

and

$$h_c = 2^{-2/3}(F_H^{2/3} - F_L^{2/3}). \qquad [20]$$

Also, for a control point to exist,

$$H_c > 1. \qquad [21]$$

Now the energy equations [3] and [4] remain valid if written between a station far upstream, where velocities are vanishingly small and $h = h_1$, and the control point. Following the same procedure as used to obtain [6] but noting that $u_{sc} H_c = u_{s2}$, it is found that

$$\frac{F_H^2}{\left(\frac{H_c}{2} + h_c\right)^2} - \frac{F_L^2}{\left(\frac{H_c}{2} - h_c\right)^2} = 2(h_1 - h_c). \qquad [22]$$

F_H and F_L can be eliminated from [22] by using [17] and [18] to find that

$$h_1 = \tfrac{3}{2} h_c; \qquad [23]$$

which is precisely the result obtained by Craya (1949) for the incipient induction of one phase by the other when discharging from a reservoir through an offtake in a vertical wall.

One can also use [19] and [20] to eliminate H_c and h_c from [22] to find that

$$h_1 = \frac{3}{2^{5/3}}(F_H^{2/3} - F_L^{2/3}). \qquad [24]$$

This is the required relationship between the level far upstream and the flowrates. Note that the level h_1' is independent of the offtake size H_2', as is expected.

It is of interest to examine [24] for the case when one phase is just inducing flow of the other phase. Then

$$\left(\frac{\Delta\rho g}{\rho}\right)^{1/3} \frac{h_1'}{Q^{2/3}} = \frac{3}{2^{5/3}} = 0.945. \qquad [25]$$

Craya (1949), theoretically, and Gariel (1949), experimentally, found that the coefficient on the right of [25] was 0.759 for the case of discharge from a reservoir through an offtake in a vertical wall. Equation [25] overpredicts h_1' by about 25% when applied to the system studied by Craya.

Equivalent results for the axisymmetric system, derived in the appendix, are:

$$h_1 = \tfrac{5}{4} h_c \qquad [26]$$

$$D_c = 1.32(F_H^{0.84} + F_L^{0.84})^{1/2.1} \qquad [27]$$

which is accurate to $\pm 0.5\%$; and

$$h_1 = 0.862(F_H^{0.4} - F_L^{0.4}) \qquad [28]$$

which is accurate to $\pm 5\%$.

When entrainment of one phase by the other is incipient, the accurate result of the theory is

$$h_1 = 0.825\, F^{0.4} \qquad [29]$$

which reduces to

$$\left(\frac{\Delta\rho g}{\rho}\right)^{0.2} \frac{h_1'}{Q^{0.4}} = 0.908. \qquad [30]$$

The value of h_1' predicted by [30] is 32% higher than found theoretically by Craya (1949) and experimentally by Gariel (1949) and Smoglie & Reimann (1986) for the incipient entrainment of water in a reservoir by air discharging through an offtake in a vertical wall. It is 21% higher than that found experimentally by Smoglie & Reimann for the incipient entrainment of air by water.

6. EXPERIMENTAL RESULTS

6.1. Zero air flowrate

The basic hydraulic engineers theory was tested by operating the rig with water flow only and using the square-cut and bell-mouth entries. To obtain agreement between the theory and experiment F_H had to be multiplied by a coefficient of discharge which was 0.80 for the square-cut entry and 0.88 for the bell-mouth. These are reasonable values (Chow 1981).

6.2. *With air flowrate*

Experiments were carried out with square-cut and bell-mouth entries. The values of h_l calculated from [28] for calculated values of $D_c > 1$ are plotted against experimental values in figures 8 and 9. The agreement between theory and experiment is remarkably good and no adjustment of the theory is necessary. It is noted that F_H was set at values of approx. 0.1, 0.2, 0.4 and the maximum attainable value of about 0.65. The air flowrate was varied to give values of F_L from 0 up to the maximum attainable value of about 0.5.

Data for which the calculated values of $D_c < 1$ are not plotted in figures 8 and 9. However, h_l was then calculated from the extension of the hydraulic engineers theory and, for zero air flowrate, exhibited the error corresponding to the coefficient of discharges given in the last subsection. As the air flowrate increased and $D_c \rightarrow 1$ the agreement between experiment and theory improved.

Smoglie & Reimann's (1986) data were taken from Smoglie's (1984) thesis. Only the data for offtakes of 6 and 20 mm dia are plotted in figure 10 as calculated vs experimental values of h_l. Data for a 12 mm offtake are not shown to avoid confusion but, if plotted, they would lie between the data for the other two offtakes in figure 10.

The agreement between theory and experiment for the 20 mm offtake is as good as for the 84 mm pipe shown in figures 8 and 9. The data for the 6 mm tube lie an almost uniform distance from the 45° correlating line through the origin. If the observed discrepancy is real it must be due to some special property of one of the phases, which does not occur in the theory. Otherwise one would expect a divergence of data to one side of the correlating line on one side of the origin to be mirrored by an equal divergence on the other side of the line on the other side of the origin. The property would have to occur in some dimensionless group involving the offtake diameter and this view is reinforced by noting that the calculated range of D_c for the data in figures 8 and 9 was 1–1.49, for the 20 mm offtake it was 1.34–3.76 and for the 6 mm offtake it was 4.22–5.79. The ranges of the 20 and 6 mm tube almost overlap and it is improbable that D_c could be used as a correlating parameter to bring all the data in figure 10 together.

Another explanation for the difference between experiment and theory, which is suggested by the almost constant offset of the data points for the 6 mm offtake from the 45° correlation line,

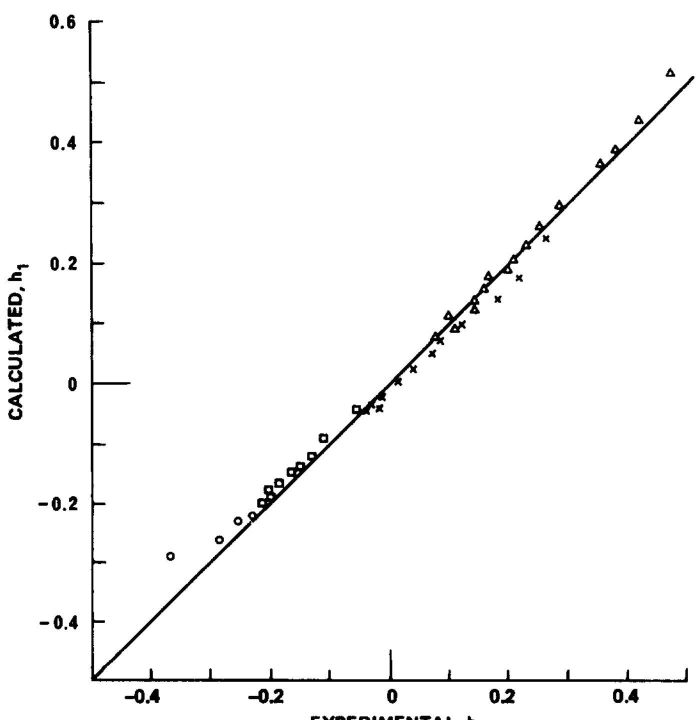

Figure 8. Comparison between experimental and calculated reservoir levels—84 mm bore tube with square-cut entry. F_H-values: ○, 0.097; □, 0.206; ×, 0.412; △, 0.677.

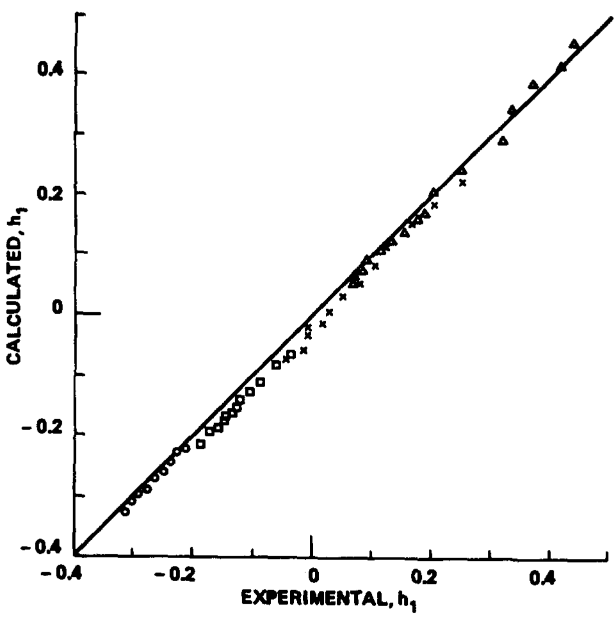

Figure 9. Comparison between experimental and calculated reservoir levels—84 mm bore tube with bell-mouth entry. F_H-values: ○, 0.091; □, 0.193; ×, 0.396; △, 0.647.

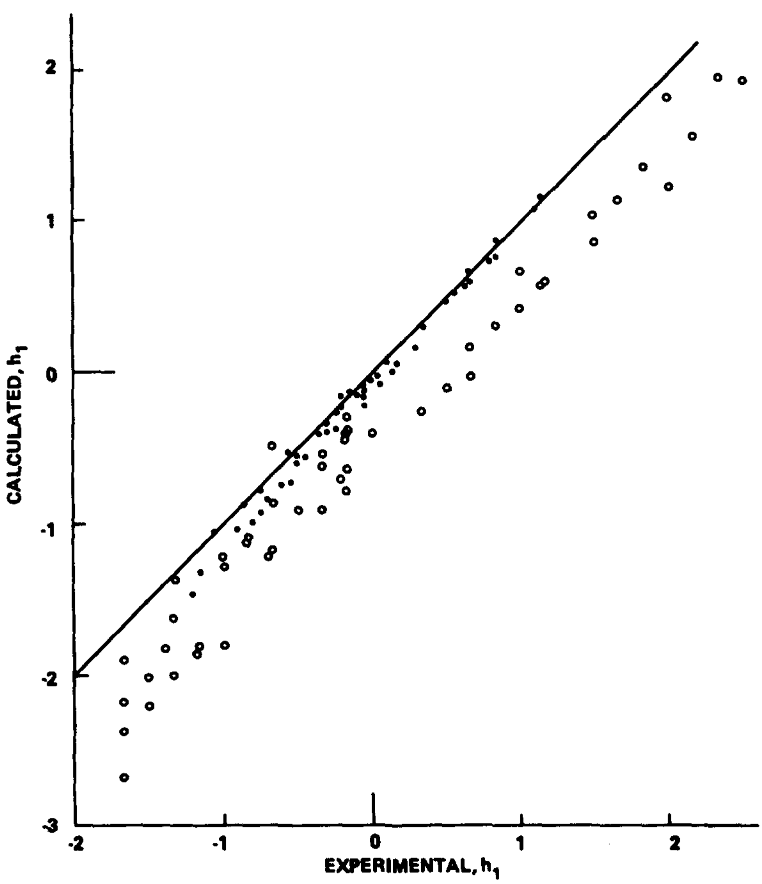

Figure 10. Comparison between experimental and calculated reservoir levels—data of Smoglie (1984). Offtake diameter: ○, 6 mm; ●, 20 mm.

is that there was a constant bias in the measurement of water level. The reservoir was a 206 mm horizontal tube and the level was measured by a scale set outside the tube. It was reported by Smoglie (1984) that the measurement error was ± 1 mm with a smooth interface and ± 5 mm with a wavy interface. The average difference between the experimental results and theory was a Δh of 0.450, 0.181 and 0.077 for the 6, 12 and 20 mm offtakes, respectively. These values are equivalent to biased level measurements of 2.7, 2.2 and 1.5 mm, respectively.

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APPENDIX

Theory for an Axisymmetric System with a Control Point

The circular cross-section of the converging axisymmetric system with its axis horizontal is shown in figure A.1. The whole cross-sectional area in A', the cross-sectional area occupied by the heavy phase is A'_H, the diameter is D' and the height of the interface above the axis is h'. The angle θ is also defined in figure A.1.

The differential energy [12] and [13] still apply but the constant volumetric fluxes are

$$Q_H = u_H A'_H, \quad Q_c = u_L(A' - A'_H). \qquad [A.1]$$

The pressure gradient is eliminated between [12] and [13], and [A.1] are used to eliminate velocities. Manipulating the result gives

$$-\frac{F_H^2}{A'A^3D^4}\frac{dA'_H}{dX} - \frac{F_L^2}{A'(1-A)^3D^4}\left(\frac{dA'_H}{dX} - \frac{dA'}{dX}\right) + \frac{dh}{dX} = 0 \qquad [A.2]$$

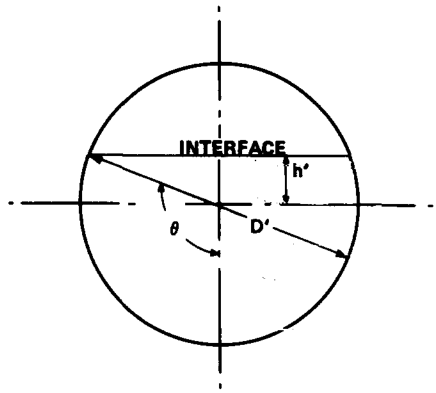

Figure A.1. Cross-section of the converging tube.

where

$$A = \frac{A'_{\mathrm{H}}}{A'}, \quad D = \frac{D'}{D'_2}, \quad X = \frac{x}{D'_2}. \qquad [A.3]$$

Now,

$$A'_{\mathrm{H}} = \frac{D'^2}{4}(\theta - \sin\theta\cos\theta) \qquad [A.4]$$

and

$$\frac{h'}{D'} = -\frac{\cos\theta}{2}. \qquad [A.5]$$

Therefore,

$$\frac{\mathrm{d}A'_{\mathrm{H}}}{\mathrm{d}X} = \frac{2A'_{\mathrm{H}}}{D'}\frac{\mathrm{d}D'}{\mathrm{d}X} + \frac{D'^2}{2}\sin^2\theta\frac{\mathrm{d}\theta}{\mathrm{d}X}. \qquad [A.6]$$

Also,

$$\frac{\mathrm{d}\theta}{\mathrm{d}X} = \frac{2}{\sin\theta}\left(\frac{1}{D'}\frac{\mathrm{d}h'}{\mathrm{d}X} - \frac{h'}{D'^2}\frac{\mathrm{d}D'}{\mathrm{d}X}\right) \qquad [A.7]$$

and

$$\frac{\mathrm{d}A'}{\mathrm{d}X} = \frac{\pi D'}{2}\frac{\mathrm{d}D'}{\mathrm{d}X}. \qquad [A.8]$$

Making substitutions into [A.2],

$$\left[\frac{\pi D^5}{4\sin\theta} - \frac{F_{\mathrm{H}}^2}{A^3} - \frac{F_{\mathrm{L}}^2}{(1-A)^3}\right]\frac{\mathrm{d}h}{\mathrm{d}X} = \left[\frac{F_{\mathrm{H}}^2}{A^3}\left(\frac{\pi}{2}\frac{A}{\sin\theta} - \frac{h}{D}\right) - \frac{F_{\mathrm{L}}^2}{(1-A)^3}\left[\frac{\pi(1-A)}{2\sin\theta} + \frac{h}{D}\right]\right]\frac{\mathrm{d}D}{\mathrm{d}X}. \qquad [A.9]$$

It follows that the two equations for the control point are

$$\frac{\pi D_{\mathrm{c}}^5}{4\sin\theta_{\mathrm{c}}} - \frac{F_{\mathrm{H}}^2}{A_{\mathrm{c}}^3} - \frac{F_{\mathrm{L}}^2}{(1-A_{\mathrm{c}})^3} = 0 \qquad [A.10]$$

and

$$\frac{\theta_{\mathrm{c}}F_{\mathrm{H}}^2}{A_{\mathrm{c}}^3} = \frac{(\pi-\theta_{\mathrm{c}})F_{\mathrm{L}}^2}{(1-A_{\mathrm{c}})^3}. \qquad [A.11]$$

From [A.10] and [A.11],

$$\frac{F_{\mathrm{H}}^2}{A_{\mathrm{c}}^3} = \frac{(\pi-\theta_{\mathrm{c}})D_{\mathrm{c}}^5}{4\sin\theta_{\mathrm{c}}} \qquad [A.12]$$

and

$$\frac{F_L^2}{(1-A_c)^3} = \frac{\theta_c D_c^5}{4 \sin \theta_c}. \qquad [A.13]$$

Next consider energy equations [3] and [4], which remain valid but draw them up for a station far upstream and the control point. Eliminate pressures between the equations, use [A.1] to eliminate velocities and then manipulate the result to obtain

$$\frac{F_H^2}{A_c^2 D_c^4} - \frac{F_L^2}{(1-A_c)^2 D_c^4} = 2(h_1 - h_c). \qquad [A.14]$$

Using [A.12] and [A.13] to eliminate F_H and F_L in [A.14] and then manipulating the result gives

$$h_1 = \tfrac{5}{4} h_c, \qquad [A.15]$$

which is identical with Craya's (1949) theoretical result for the incipient entrainment of one phase by the other in discharging from a reservoir through an offtake in a vertical wall.

Now

$$D_c = \frac{D_c'}{D_2'} = -\frac{2h_c'}{D_2' \cos \theta_c} = -\frac{8}{5}\frac{h_1}{\cos \theta_c} \qquad [A.16]$$

so that [A.12] and [A.13] become

$$\frac{F_H^2}{h_1^5} = -\frac{1}{4}\left(\frac{8}{5}\right)^5 \frac{(\pi - \theta_c)A_c^3}{\sin \theta_c \cos^5 \theta_c} \qquad [A.17]$$

and

$$\frac{F_L^2}{h_1^5} = -\frac{1}{4}\left(\frac{8}{5}\right)^5 \frac{\theta_c(1-A_c)^3}{\sin \theta_c \cos^5 \theta_c} \qquad [A.18]$$

and the problem is to eliminate the parameter θ_c between the last two equations. Using the experience of the two-dimensional theory and noting that h_1' should not depend upon D_2', we could seek a series solution for h_1 composed of terms $(F_H^n - F_L^n)^{0.4/n}$, where n is varied. In the event it is found that

$$h_1 = 0.862\,(F_H^{0.4} - F_L^{0.4}) \qquad [A.21]$$

is accurate to $\pm 5\%$.

It is also found by trial and error from [A.12] and [A.13] that

$$D_c = 1.32[F_H^{0.84} + F_L^{0.84}]^{1/2.1} \qquad [A.22]$$

is accurate to $\pm 0.5\%$.